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Isomorphism Property Of Adjacent Graphs For Some Finite Groups

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Abstract: In this paper, Adjacent Graphs for some finite groups were created, which is based on Algebraic Graph Theory. For this Adjacent Graph Isomorphism property is applied and proved using suitable theorems. For the theorems Algebraic methods are used and described with Graphs as in Algebraic Graph Theory, most Algebraic Methods are solved using Graphs. As Graph Theory is a link between lines and points, the Adjacent Graph for some finite Groups is evolved. Also the relationship between Order of the Group and the Adjacent Graph were established. The connections between non-self-invertible elements of the Group with the points of Adjacent Graphs is also derived. Some properties related to Cycle with the Groups and the isomorphic property to Group and Adjacent Graph are discussed with some conditions. Appropriate examples for the Adjacent Graphs were also included.

Keywords: Cycle, Finite Abelian Group, Finite Group, Graph, Planar Graph**.**

AMS Subject Classification: 05C38, 20K01, 20D05, 05C25, 05C10.

1. Introduction

In Mathematics field, Graph theory is the learning of Graphs that means the tie between points and lines as vertices and edges. A Graph is a graphic description of a group of things where pair of things are merged by links. Graph Theory are also applied in Computer Science, Electrical Engineering, Physics and Chemistry.

A Group is an assembly of components or objects that are merged together to perform some operations on them. The Study of a set of elements in a group is called Group Theory. An idea of Group is derived from Abstract Algebra. Rings, Fields and Vector Spaces are some familiar Algebraic Structures to be considered as groups with some additional operations and axioms. Whenever an object's property is same, the object can be examined under Group Theory being

Group Theory is the study of Symmetry, The method to solve Rubik's cube also acts based on Group Theory.

Algebraic Graph Theory is a branch of Mathematics that involves to find the solutions for Algebraic Methods by using Graph Theory concepts. Linear Algebra, Group Theory and the study of Graph invariants are three main divisions of Algebraic Graph Theory. There is a bind between Graph Theory and Group Theory, which is shown by Arthur Cayley. He was the first to introduce the Cayley Graphs to finite groups.

The main aim of this paper is to derived some properties of Adjacent Graph for Groups related order of the Group and Isomorphic property. Few Graphs are given as examples for Adjacent Graphs and for theorems.

2. Preliminaries

Definition 2.1

The **Order of the Group G** is defined by the cardinality of G and is written by | G |.

Definition 2.2

The **Order of the Graph X** is the cardinality of its vertex set. It is denoted by O(X).

Definition 2.3

First Theorem of Graph Theory

In a Graph X, The sum of the degrees of the vertices is equal to twice the number of edges.

Definition 2.4

The function is **surjective** if f(A) = B. The function is also called as onto function.

Definition 2.5

The function is **injective** if $f(a) = f(a') \Rightarrow a = a'$. The function is also called as one-to-one function.

Definition 2.6

The function is **bijective** if it is both injective and surjective.

Definition 2.7

Let $(G, \circ)$ and $(H, *)$ be two groups. These groups are said to be isomorphic if there is a bijective map $\varphi : G \square H$, such that $\varphi(x \circ y) = \varphi(x) * \varphi(y)$ for all $x, y \in G$. Such a map φ is called an **Isomorphism.** When such a map exists, we write $G \cong H$.

Definition 2.8

A **Cycle** is a closed trail in which the first vertex is equal to the last vertex where only vertex is repeated.

Definition 2.9

Let (G,∗) be a finite group and I = {g ∈ G / g ≠ g^{-1}}. We define the **Adjacent Graph $A_I(G)$** associated with G as graph whose set of vertices coincides with G such that two distinct vertices g and h are adjacent if and only if g ∗ h ∈ I or h ∗ g ∈ I.

Example

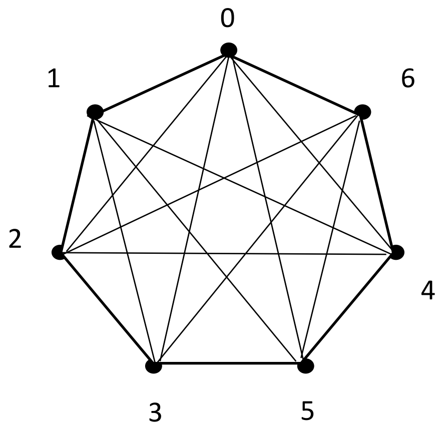

Figure Adjacent Graph for the group (Z_7 , +)

Result 2.1

The Adjacent graph $A_I(G)$ is not complete for every finite Group G.

3. Isomorphism of Adjacent Graphs

Theorem 3.1

Let G be a group with O(G) = 3. Then $A_I(G)$ is not a cycle.

Proof:

Given G is a finite group with O(G) = 3

Let $A_I(G)$ be the adjacent graph for the finite group G.

The only non-empty adjacent graph of order 3 is P_2, the path of length 2.

Which is not a cycle.

Let us take G = (Z_3 , +)

(i.e.) G = {0, 1, 2} Here n = 3

I = {1, 2}

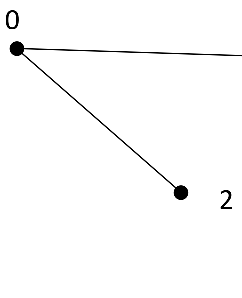

Figure Path P_2

which is the path of length 2.

Theorem 3.2

Let G be a group with O(G) = 4 and $A_I(G)$ be a non-empty graph. Then $G \cong Z_4$.

Proof:

Given G is a group and O(G) = 4.

The Adjacent graph $A_I(G)$ is non empty graph.

Let I be the set of non-self-invertible elements of G.

Since the elements of I occurs in pairs, |I| = 0 or 2 or 4

We know that, $|I| \neq 0$ $[\because A_I(G) \neq \varphi]$

And also, $|I| \neq 4$ $[\because |G| = 4 \text{ and } e \notin I]$

Therefore, $|I| = 2$

Hence G has exactly one non-trivial self-invertible element.

Therefore $G \cong Z_4$.

Theorem 3.3

Let G be a finite group with O(G) > 4. Then $A_I(G)$ is the cycle C_4 if and only if $G \cong Z_4$.

Proof:

Given G is a finite group with O(G) > 4

Let $A_I(G)$ be the adjacent graph for the finite group G.

First Assume that $A_I(G) = C_4$

To Prove: $G \cong Z_4$.

By Contrary Suppose $G \ncong Z_4$

If $n \geq 5$

Since the group G is a cycle with 4 vertices, each vertices of the adjacent graph $A_I(G)$ has degree 2.

We know that, In any adjacent graph deg (e) = | I |= 2

Therefore, 2 elements a and b $\in$ G are non-self-invertible elements.

Since the identity element e is adjacent to both a and b, there should be a self-invertible element c which is adjacent to a.

Hence, a * c = b and b * c = a

Thus c is adjacent to both a and b

But we have $n \geq 5$, Then $A_I(G)$ to be disconnected.

Therefore which is not cycle.

Our assumption is wrong,

Therefore the only possible is n = 4

Hence by the Theorem-3.2, $G \cong Z_4$

Conversely assume that $G \cong Z_4$.

Construct the graph $A_I(Z_4)$

Let us consider the group $G = (Z_4 , +)$

Then the adjacent graph $A_I(Z_4)$ is obvious the Cycle C_4.

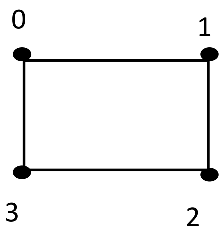

Figure Cycle C_4

Hence the Proof.

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